Csc 225 a4

Saturday, November 9, 2024 11:26 AM

Q1)

```
function longestpath (G, s, t)

    foreach Vertex V in G
        distance[v] = ∞
        parent[v] = null
    distance[s] = 0

    #Get topological order
        Order = topological_Sort(G)

    #Get length of edges
    for ed in order
        if distance[u] != ∞
            foreach (v, weight) in G[u]
                if distance[v] < distance[u] + weight
                    distance[v] = distance[u] + weight
                    parent[v] = u

    #Build the longest path
    if distance != ∞
        path = []
        Current = t
        while current is not null
            Path.append(current)
            current = parent(current)
        path.reverse()
        return path, distance[t]
    else
        return "not connected"
```

∴ finds max length using a adjacency list with a time complexity of $O(M+N)$ due to topological sort

Q2)

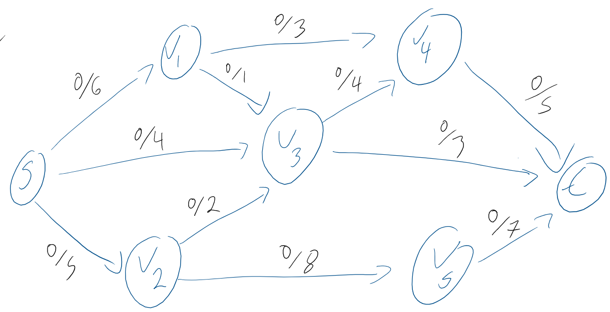
From the given statment we can infer

i, j is the connected edges with there respive weights

for $M^2(i,j)$ repersends the shortest path from $i \to j$ using a max of 1 intermidet node

for $M^k(i,j)$ we find that it is the shorest path overall in the graph

Q3


s
v_1
v_2
v_3
v_4
v_5
t
0/6
0/4
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0/1
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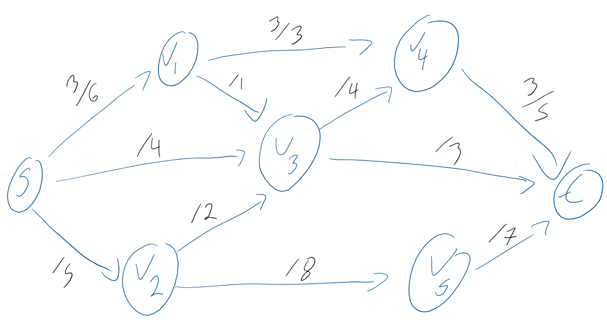
s
v_1
v_2
v_3
v_4
v_5
t
3/6
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/2
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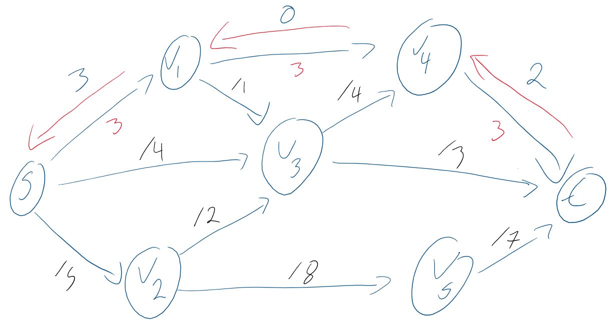
s
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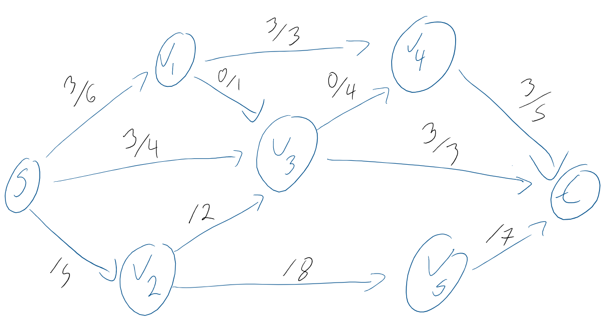
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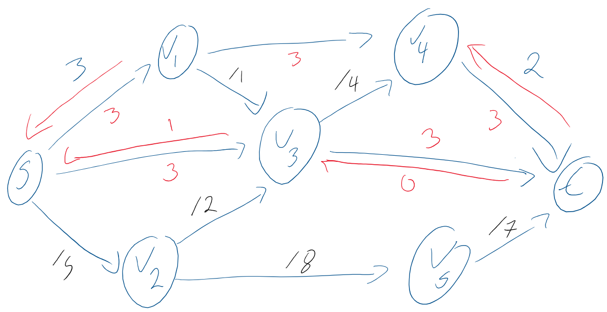
s
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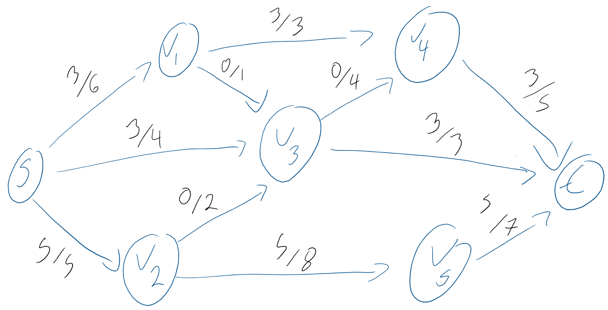

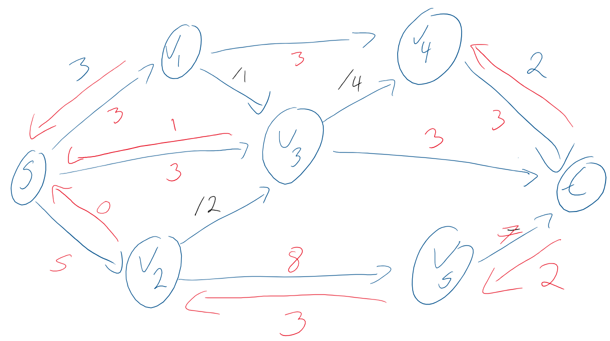

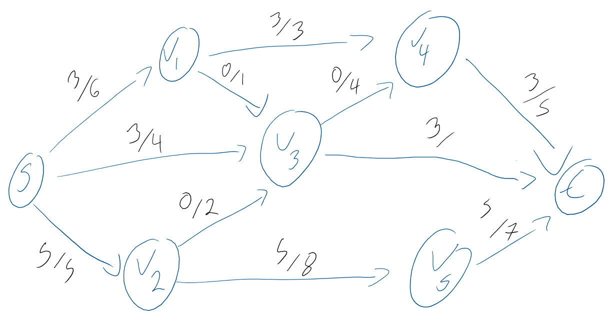

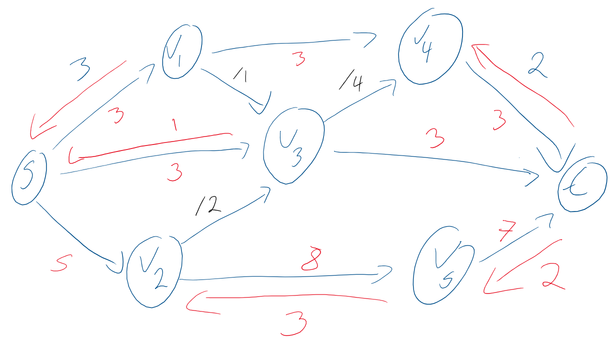

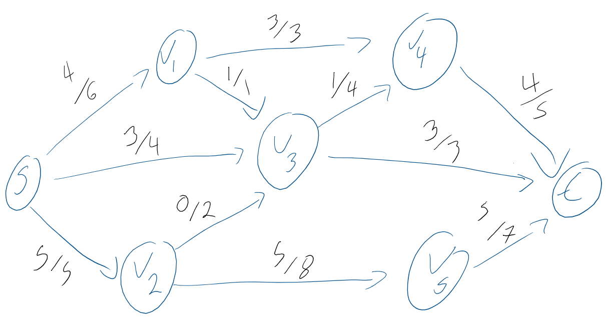

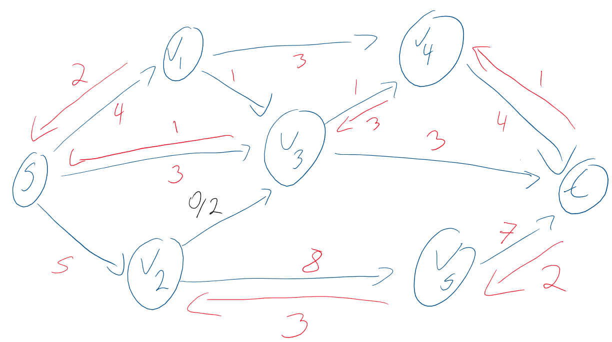

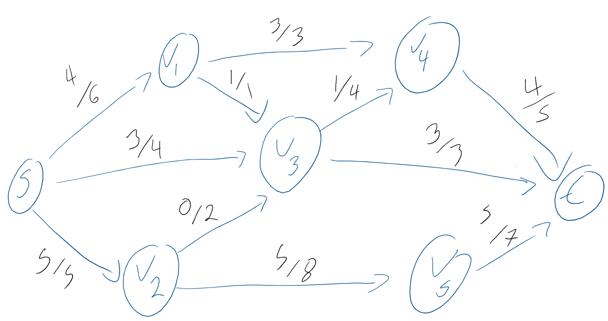

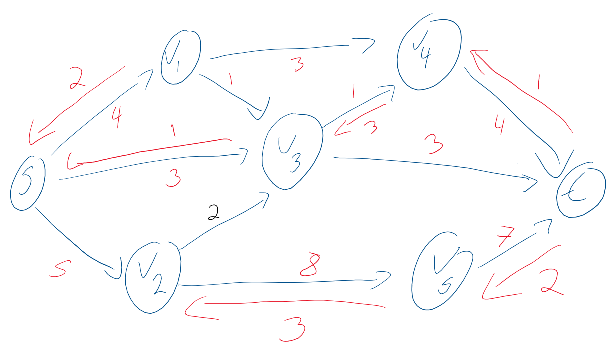

∴ Flow G = 12

st-Cut = 6

$s \xrightarrow[3]{} v_3 \xrightarrow[3]{} t$

Q4)

Base Case

When $|S| = 1$, the Set S Contains Vertex s,
Such that the only element is its self

so $S = \{s\}$, $T = V / \{s\}$ f $s \in S$, $t \in T$

deff of Cut

$$f(x) = \sum f(u,v) - \sum f(u,v)$$

Since s is a Sink no flow can come into s

$$f(x) = \sum_{v \in T} f(s,v)$$

$\therefore$ by property of flow in = out
the Sum is equal to $|f|$.

$\therefore$ the flow of $s \to t$ is the total flow and repersents the flow leaving s

$\therefore$ $f(x) = |f|$

IS

Assume for any s-t Cut $x = (S,T)$ with $|S| = k$ with the flow accross the cut being the total flow $|f|$

for some s-t Cut $x' = (S', T')$ for $|S'| = k+1$, $S' = S \cup \{u\}$
for some $u \in T$.

flow for x'

$$f(x') = \left(\sum_{v \in S'} f(v,w) - \sum_{v \in T'} f(v,w)\right) + \left(\sum_{w \in T} f(v,w) - \sum_{w \in T} f(v,w)\right)$$

$$= f(x) + \sum_{w \in T} f(v,w) - \sum f(v,w)$$

by flow Conservation (in = out)

$$\sum_{u \in T} f(u,v) = \sum_{v \in T} f(v,u)$$

$\therefore f(x') = f(x)$

$\therefore f(x') = f(x) = |f|$

Conclusion

by induction $|f| = f(x)$ is true
useing the cut property and flow conseruction